FRACTIONS 3
STUDENT PACKET

FRACTION MULTIPLICATION AND DIVISION

Parent (or Guardian) signature ______________________________________

MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. (See section 3.5.) Key mathematical vocabulary is underlined throughout the packet.

commutative property of multiplication	distributive property
product	quotient
reciprocal	word of your choice: _________________

OPENING PROBLEM: THE COOKIE JAR

There are some cookies in a jar. Julia eats $\frac{1}{2}$ of them. Rhianna then eats $\frac{1}{3}$ of the remaining cookies. Kaelen then eats $\frac{1}{4}$ of the remaining cookies. Lastly, Ally eats one cookie. If there are two cookies left, how many were there at the beginning?

FRACTION MULTIPLICATION

We will use pictures and procedures to multiply fractions.

GETTING STARTED

1. Draw 3 groups, each having 5 dots.

How many total dots is this? _____

2. Draw 5 groups, each having 3 dots.

How many total dots is this? _____

3. How does the commutative property of multiplication relate to problems 1 and 2?

Find this term in section 3.5 and record it in My Word Bank.

4. Rewrite 4 • 6 as a repeated addition expression. ____________________________

5. Rewrite 6 • 4 as a repeated addition expression. ____________________________

6. Compute mentally.

a. $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \boxed{}$ This is _____ groups (or copies) of $\boxed{}$.

b. $1\frac{1}{4} + 1\frac{1}{4} + 1\frac{1}{4} = \boxed{}$ This is _____ groups (or copies) of $\boxed{}$.

FOOD FRACTIONS 1

Follow your teacher's directions to explore some fraction problems.

(1)	(2)
(3)	(4)

PRACTICE 1

Fill in the table. Some of the diagrams have been started for you.

	Verbal Interpretation	Multiplication Expression	Diagram	Product
1.	3 groups of $\frac{1}{6}$			
2.		$\frac{1}{6}(3)$		
3.	4 groups of $\frac{2}{3}$			
4.		$\frac{3}{4}(6)$		

5. Explain how problems 1 and 2 connect to the commutative property of multiplication.

6. Agnes said that the product in problem 3 is "eight over three." Write in words a more precise way to express this fraction's value.

Compute.

7. $4\left(\frac{2}{9}\right)$	8. $10\left(\frac{7}{11}\right)$	9. $\frac{3}{10}(100)$

10. Describe in words how to multiply a whole number times a proper fraction.

FOOD FRACTIONS 2

Follow your teacher's directions to explore some fraction problems.

(1)

(2)

(3-4)

PRACTICE 2

Fill in the table and find the products of these proper fractions.

	Multiplication Expression	Verbal Interpretation	Diagram	Product
1.	$\frac{1}{2} \bullet \frac{2}{3}$			
2.		$\frac{1}{3}$ of a group of $\frac{2}{3}$		
3.	$\frac{1}{2}\left(\frac{1}{6}\right)$			
4.		$\frac{1}{4}$ of a group of $\frac{3}{4}$		
5.	$\frac{2}{3} \times \frac{3}{4}$			
6.		$\frac{2}{5}$ of a group of $\frac{2}{3}$		

7. Write a short story that can be represented by $\frac{1}{2} \bullet \frac{2}{5} = \frac{2}{10}$.

8. Write the multiply-across rule for fraction multiplication. It can be found in section 3.5.

Words:

Symbols: $\frac{a}{b} \bullet \frac{c}{d} =$ ______ $(b \neq 0,\ d \neq 0)$

A FRACTION MULTIPLICATION SHORTCUT

Follow your teacher's directions for (1), (2), and (3).

(1) Longer version:	(2) Shorter version: (3) Now try the "yikes" problem:

4. Explain Taylor's work: $\frac{\cancel{18}^{\,2}}{{}_{11}\cancel{55}} \bullet \frac{\cancel{20}^{\,4}}{\cancel{27}_{\,3}} = \frac{8}{33}$

Compute. Use any strategy.

5. $5\left(\frac{1}{6}\right)$	6. $\frac{1}{7} \times \frac{3}{8}$	7. $\frac{2}{5} \bullet \frac{5}{6}$
8. $\left(\frac{4}{16}\right)\left(\frac{3}{9}\right)$	9. $6 \bullet \frac{2}{5}$	10. $\frac{2}{9} \bullet \frac{4}{7}$
11. $\frac{3}{4} \bullet 20$	12. $\frac{6}{21} \times \frac{14}{24}$	13. $\frac{4}{5}(9)$

MULTIPLYING MIXED NUMBERS

Follow your teacher's directions for problems 1-5.

(1) ______ people each eat ______ slices of toast.

Picture:

There are ______ **whole** slices of toast in all.

Circle the step in your work to the right that illustrates using the distributive property. Complete an explanation and example in My Word Bank.

Work:

(2)	(3)	(4)
(5)		

Compute.

6. $\left(2\frac{2}{3}\right)\left(4\frac{1}{2}\right)$	7. $2 \bullet 1\frac{1}{9}$
8. $2\frac{2}{7} \bullet 10\frac{1}{2}$	9. $1\frac{2}{5} \times 1\frac{1}{14}$

PRACTICE 3

Find the product of $\frac{1}{3} \bullet \frac{2}{3}$ by drawing an area model diagram. ______

1. The multiply-across rule for fractions states that $\frac{a}{b} \bullet \frac{c}{d} = \underline{\qquad}$.

2. Explain Ryan's work: $3\frac{3}{4} \bullet 12 = \frac{15}{{}_1\cancel{4}} \bullet \frac{\cancel{12}^{3}}{1} = \frac{45}{1} = 45$

Compute.

3. $\frac{2}{9}\left(\frac{3}{8}\right)$	4. $\frac{8}{9} \bullet \frac{6}{24}$	5. $6 \times 4\frac{1}{3}$
6. $2 \bullet 5\frac{3}{5}$	7. $\left(3\frac{3}{5}\right)\left(1\frac{1}{9}\right)$	8. $6\frac{2}{3} \bullet 1\frac{1}{5}$
9. $\frac{2}{3} + \frac{4}{5}$	10. $\frac{3}{4} - \frac{1}{3}$	11. $1\frac{5}{16} \bullet 3\frac{3}{7}$

12. In the opening Cookie Jar Problem, we found that there were 12 cookies at the beginning. Kaelen has $\frac{1}{4}$ of $\frac{4}{12}$ of those 12 cookies. Write a multiplication expression to represent this. Then compute the number of cookies Kaelen has based on this expression.

FRACTION DIVISION: DIVIDE-ACROSS

We will use pictures and procedures to divide fractions.

GETTING STARTED

Compute.

1. $\frac{2}{5} + \frac{3}{4}$	2. $\frac{8}{9} - \frac{5}{6}$	3. $\frac{5}{8} \bullet \frac{4}{5}$

4. Use your knowledge of fraction multiplication to fill in the blank. $\boxed{} \bullet \frac{3}{11} = \frac{6}{55}$

Simplify. Show work.

5. $\frac{7}{35}$	6. $\frac{18}{30}$

7. Find quotient in section 3.5 and write an explanation and example in My Word Bank.

8. Circle all of the expressions below that could represent 6 ÷ 3.

   a. $\frac{6}{3}$

   b. How many 3's go into 6?

   c. $\frac{3}{6}$

   d. How many 3's does it take to make 6?

   e. How many 6's are in 3?

   f. How many groups of 3 are there in 6?

9. Write a short story that can be represented by 6 ÷ 3 = 2.

EXPLORING DIVIDE-ACROSS

Follow your teacher's directions to explore fraction division problems 1-4.

Words	Diagram	Division Statement	Quotient
(1) How many groups of $\square$ are in $\square$?			
(2) How many groups of $\frac{\square}{\square}$ are in $\frac{\square}{\square}$?			
(3) How many groups of $\square$ are in $\square$?			
(4) How many groups of $\frac{\square}{\square}$ are in $\frac{\square}{\square}$?			

Use your knowledge of the relationship between multiplication and division to fill in the blanks. For each problem, the same number must go into $\square$ or $\bigcirc$.

	Multiplication Problem	Related Division Problem	Divide Across	Equal Quotients?
5.	$\square \bullet 4 = 8$	$8 \div 4 = \square$		
6.	$\frac{\square}{\bigcirc} \bullet \frac{4}{10} = \frac{8}{10}$	$\frac{8}{10} \div \frac{4}{10} = \frac{\square}{\bigcirc}$	$\frac{8 \div 4}{10 \div 10} = \frac{\square}{\bigcirc}$	
7.	$\frac{\square}{\bigcirc} \bullet \frac{5}{5} = \frac{5}{10}$	$\frac{5}{10} \div \frac{5}{5} = \frac{\square}{\bigcirc}$	$\frac{5 \div 5}{10 \div 5} = \frac{\square}{\bigcirc}$	
8.	$\frac{\square}{\bigcirc} \bullet \frac{4}{3} = \frac{8}{15}$	$\frac{8}{15} \div \frac{4}{3} = \frac{\square}{\bigcirc}$	$\frac{8 \div 4}{15 \div 3} = \frac{\square}{\bigcirc}$	

9. Does it appear that dividing across works?

THE DIVIDE-ACROSS RULE

Follow your teacher's directions.

(1) – (3) Connor eats _____ of a small cake. A serving is _____ cake. How many servings does Connor eat? How many servings of $\frac{\square}{\square}$ are in $\frac{\square}{\square}$? Division Problem: Diagram: Computation: Answer question:	(4) – (6) Mia eats _____ cup of cereal. A serving size is _____ cup. How many servings does Mia eat? How many servings of $\frac{\square}{\square}$ are in $\frac{\square}{\square}$? Division Problem: Diagram: Computation: Answer question:

(7) What is the divide-across rule for fractions?

Words:

Symbols:

(8)	(9)

PRACTICE 4

1. Chase has $1\frac{1}{2}$ sandwiches leftover from yesterday's party. A serving size is $\frac{3}{4}$ of a sandwich. How many servings does he have?

   Represent this situation with a picture and a division expression.
   Then perform the divide-across procedure.
   Clearly show your work, and the result.

Compute.

2. $\frac{1}{3} \div \frac{5}{9}$	3. $\frac{1}{2} \div \frac{3}{5}$	4. $2\frac{1}{8} \div \frac{3}{4}$
5. $1\frac{3}{4} \div \frac{1}{2}$	6. $1\frac{1}{8} \div 4\frac{1}{2}$	7. $\frac{1}{2} \div 4$

8. Andrea tried to calculate $2\frac{2}{3} \div \frac{4}{5}$ as illustrated below and got stuck.

$$2\frac{2}{3} \div \frac{4}{5} = \frac{8}{3} \div \frac{4}{5} = \frac{2}{\frac{3}{5}}$$

   Even though she did nothing wrong, show a different approach that might be more successful for her.

PRACTICE 5

1. A 2-foot-long sandwich is cut into portions that are $\frac{3}{4}$ feet long each. a. Write a division expression that represents this situation. Words: Numbers: b. Use a diagram to show the full portions that can be cut and any leftover part. c. Solve using the divide-across rule. d. How many full portions can be cut? e. How long is the piece that is leftover? f. What fraction of a portion is leftover? g. Check your solution by multiplication.	2. A 4-foot-long board is cut into shelves that are $1\frac{1}{4}$ feet long each. a. Write a division expression that represents this situation. Words: Numbers: b. Use a diagram to show the full shelves that can be cut and any leftover part. c. Solve using the divide-across rule. d. How many full shelves can be cut? e. How long is the piece that is leftover? f. What fraction of a shelf is leftover? g. Check your solution by multiplication.

FRACTION DIVISION: MULTIPLY-BY-THE-RECIPROCAL

We will use the inverse relationship between multiplication and division and the divide-across rule to make sense of a common fraction division rule.

GETTING STARTED

1. Find reciprocal in section 3.5 and write an explanation of it in My Word Bank.

2. Write the reciprocals of each of the following numbers.

   a. 3 b. $\frac{1}{6}$ c. $\frac{4}{5}$

3. The following pairs of numbers are reciprocals of one another. Multiply each pair of reciprocals.

   a. 5 and $\frac{1}{5}$ b. $\frac{5}{7}$ and $\frac{7}{5}$

   c. What is the result when a number is multiplied by its reciprocal? 1

4. Describe an easy way to find the reciprocal of a fraction.

5. What is the reciprocal of $\frac{a}{b}$? $\frac{b}{a}$

6. Why is $\frac{2}{3}$ the reciprocal of $1\frac{1}{2}$?

7. What is the reciprocal of $2\frac{3}{5}$?

EXPLORING MULTIPLY-BY-THE-RECIPROCAL

1. Compute.

   a. $12 \div 4$ b. $\frac{1}{4}$ of 12 c. $12 \bullet \frac{1}{4}$

   d. Does dividing by 4 and multiplying by $\frac{1}{4}$ produce the same result?

2. Compute. Use the divide-across rule in Column I and the multiply-across rule in Column II.

	Column I **Divide-across** dividend ÷ divisor = quotient	**Column II** **Multiply-across** first factor × second factor = product	**Equal Results?**
a.	$\frac{10}{21} \div \frac{2}{7}$	$\frac{10}{21} \bullet \frac{7}{2}$	
b.	$\frac{7}{8} \div \frac{1}{4}$	$\frac{7}{8} \bullet \frac{4}{1}$	
c.	$\frac{2}{3} \div \frac{1}{6}$	$\frac{2}{3} \bullet \frac{6}{1}$	
d.	$\frac{1}{6} \div \frac{2}{3}$	$\frac{1}{6} \bullet \frac{3}{2}$	

3. For each pair in problem 2 above, compare Column I and Column II.

a. How do the dividends compare to the first factors?

b. How do the divisors compare to the second factors?

c. How do the quotients compare to the products?

d. Based on these examples, it appears that dividing by a number gives the same result as multiplying by the ________________ of the ________________.

MULTIPLY-BY-THE-RECIPROCAL RULE

On the previous page you observed that dividing by a number gives the same result as multiplying-by-the-reciprocal of the divisor.

Compute. Use the divide-across rule for Column A and test the multiply-by-the-reciprocal statement for Column B.

	Column A **Divide-across**	**Column B** **Multiply-by-the-reciprocal**	**Equal Results?**
1.	$\frac{3}{4} \div \frac{5}{8}$		
2.	$\frac{2}{3} \div \frac{1}{2}$		
3.	$5 \div \frac{1}{6}$		
4.	$3\frac{1}{2} \div 4$		

5. What is the multiply-by-the-reciprocal rule for fractions? Use section 3.5 if needed.
   Words:

   Symbols:

6. Explain in words how to apply this rule to compute $3 \div 1\frac{1}{2}$.

PRACTICE 6

1. Write the reciprocal of each number.

a. 8 b. $\frac{1}{3}$ c. $\frac{5}{9}$ d. $2\frac{3}{4}$

Compute. Use the divide-across rule for Column A and the multiply-by-the-reciprocal rule for Column B.

	Column A Divide-across	Column B Multiply-by-the-reciprocal	Equal Results?
2.	$\frac{5}{6} \div \frac{1}{8}$		
3.	$3 \div \frac{2}{3}$		
4.	$1\frac{1}{4} \div 2$		

Compute using any method.

5. $\frac{9}{10} \div \frac{3}{5}$	6. $6 \div \frac{3}{4}$	7. $2\frac{1}{4} \div 1\frac{1}{6}$

8. Hector runs 3 miles around the perimeter of a park. One lap around is $\frac{2}{3}$ miles. How many full laps does he run? What fraction of a lap does he run at the end?

a. Solve with a picture.	b. Solve by computing:

c. Answer the questions.

Hector runs _____ full laps around the park and then another _____ lap.

A DIVISION PATTERN

Study the pattern that has been started below.

I Division Expression	II Quotient
$8 \div \square$	
$8 \div \square$	
$8 \div \square$	$\frac{1}{2}$
$8 \div \square$	1
$8 \div 4$	
$8 \div 2$	
$8 \div 1$	
$8 \div \frac{1}{2}$	
$8 \div \square$	32
$8 \div \square$	

1. Fill in the missing numbers.

2. Amir says, "When I divide, I think the result is always less than what we start with. Dividing makes things smaller."

   Critique Amir's reasoning.

> Recall in a division problem:
> **dividend ÷ divisor = quotient**

3. What happens to the quotient when:

   a. the dividend is divided by 1?

   b. the dividend is divided by a whole number greater than 1?

   c. the dividend is divided by a fraction between 0 and 1?

WHY DOESN'T IT BELONG?

A. $\frac{1}{3} + \frac{5}{12}$	B. $\frac{1}{2} \bullet 1\frac{1}{2}$
C. $\frac{5}{6} \div 1\frac{1}{9}$	D. $1\frac{2}{5} \div 1\frac{13}{15}$

1. Choose one expression (A-D) at a time and explain why it does not belong with the other three expressions.
   - A does not belong because…
   - B does not belong because…
   - C does not belong because…
   - D does not belong because…

2. Find the results (sum, product, and quotients) of the four expressions above.

A.	B.
C.	D.

3. Chantal said, "I think all four expressions belong." What could she have meant by this?

REVIEW

A COOKIE RECIPE

A **Deee-Lightful Chocolate Chip Cookie Recipe**

1	cup butter	$\frac{1}{2}$	teaspoon salt
1	teaspoon baking soda	$\frac{3}{4}$	cup white sugar
1	teaspoon vanilla	$\frac{2}{3}$	cup brown sugar
1	egg	$2\frac{1}{4}$	cups all purpose flour
12	ounces semi-sweet chocolate chips		

Find the amount of each item needed (units are not necessary) when the recipe is:

		Doubled	Quadrupled	Cut in Half	Cut in Thirds
1.	**eggs** (number)				
2.	**chocolate chips** (ounces)				
3.	**salt** (teaspoons)				
4.	**white sugar** (cups)				
5.	**brown sugar** (cups)				
6.	**flour** (cups)				

7. Circle one measurement that might not make sense for a given ingredient above, and explain why. What would you do about it if you were making this amount of cookies?

FRACTION DIVISION TARGETS

In each of the following problems use this format and four of the digits 1–9, no more than once each. There may be more than one correct answer for each.

$$\frac{\square}{\square} \div \frac{\square}{\square}$$

1. Find any quotient.

2. Find a quotient as close to 1 as possible.

3. Find the greatest possible quotient.

4. Find the least possible quotient.

POSTER PROBLEM

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _______.
- Each group will have a different colored marker. Our group marker is _________.

Part 2: Do the problems on the posters by following your teacher's directions.

Poster 1 (or 5)	Poster 2 (or 6)	Poster 3 (or 7)	Poster 4 (or 8)
Allie has $2\frac{1}{3}$ feet of fabric. She wants to make pillows that each require $\frac{1}{2}$ feet of fabric.	Olivia has 4 feet of fabric. She wants to make pillows that each require $\frac{3}{4}$ feet of fabric.	Monica has 5 feet of fabric. She wants to make pillows that each require $1\frac{1}{4}$ feet of fabric.	Michela has $4\frac{1}{3}$ feet of fabric. She wants to make pillows that each require $1\frac{2}{3}$ feet of fabric.

A. Copy the main facts of the problem, write a division statement, and draw a picture to represent the problem.

B. Answer: How many full pillows can be made? How long is the left over fabric? What fraction of a pillow does the leftover fabric represent?

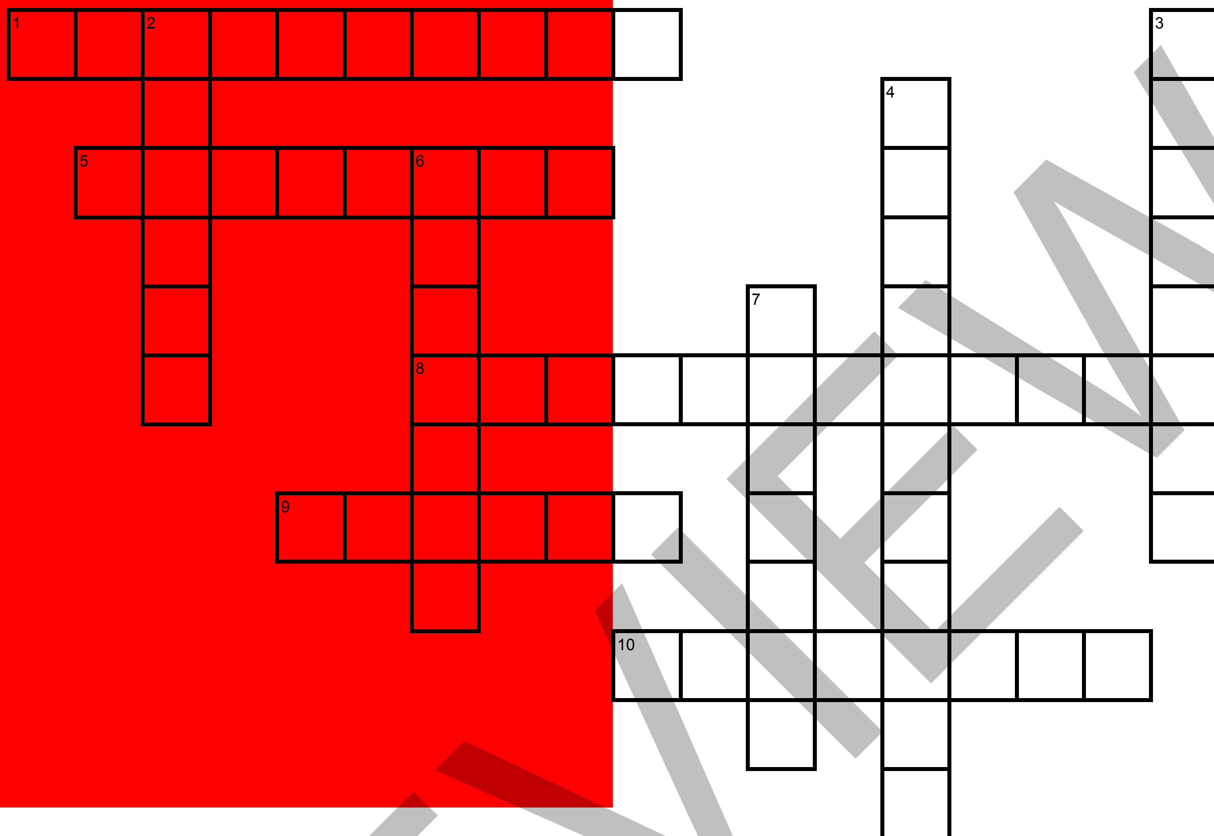
C. Compute using the divide-across rule.

D. Compute using the multiply-by-the-reciprocal rule.

Part 3: Return to your seats. Work with your group, and show all work.

Write a problem that has a mixed number for fabric length in feet, a fraction for pillow length in feet, and there is no leftover fabric.

VOCABULARY REVIEW



Across

1 To divide by a number, multiply by its ___.

5 Dividing two mixed numbers is simpler if they are changed to ___ fractions.

8 $4\left(2+\frac{1}{2}\right) = 4(2)+4\left(\frac{1}{2}\right)$ is an example of the ___ property.

9 In the equation $\frac{3}{2} \bullet \frac{1}{3} = \frac{1}{2}$, the number $\frac{1}{3}$ is a(n) ____.

10 In the equation $2 \div \frac{1}{3} = 6$, the number 6 is the ____.

Down

2 We can always divide-across with fractions that have ___ denominators.

3 In the equation $2 \div \frac{1}{3} = 6$, the number 2 is the ____.

4 The ___ property of multiplication tells us that we can multiply two numbers in any order.

6 In the equation $\frac{3}{2} \bullet \frac{1}{3} = \frac{1}{2}$, the number $\frac{1}{2}$ is the ____.

7 In the equation $2 \div \frac{1}{3} = 6$, the number $\frac{1}{3}$ is the ____.

DEFINITIONS, EXPLANATIONS, AND EXAMPLES

Word or Phrase	Definition
commutative property of multiplication	The commutative property of multiplication states that $a \bullet b = b \bullet a$ for any two numbers a and b. In other words, changing the order of the factors does not change the product. $3 \bullet 5 = 5 \bullet 3$
distributive property	The distributive property states that $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for any three numbers a, b, and c. $3(4 + 5) = 3(4) + 3(5)$ and $(2 + 7)8 = 2(8) + 7(8)$
division	Division is the mathematical operation that is inverse to multiplication. For $b \neq 0$, division by b is multiplication by the multiplicative inverse $\frac{1}{b}$ of b, $a \div b = a \bullet \frac{1}{b}$. $4 \div 3 = 4 \bullet \frac{1}{3}$ In a division problem, the number a to be divided is the dividend, the number b by which a is divided is the divisor, and the result $a \div b$ of the division is the quotient: dividend ÷ divisor = quotient $\qquad \frac{\text{dividend}}{\text{divisor}} = \text{quotient}$
product	A product is the result of multiplying two or more numbers or expressions. The numbers or expressions being multiplied to form the product are factors of the product. factor • factor = product 7 • 8 = 56.
quotient	In a division problem, the quotient is the result of the division. dividend ÷ divisor = quotient 12 ÷ 3 = 4
reciprocal	For $b \neq 0$, the reciprocal of b is the number, denoted by $\frac{1}{b}$, that satisfies $b \bullet \frac{1}{b} = 1$. The reciprocal of b is also called the multiplicative inverse of b. The reciprocal of 3 is $\frac{1}{3}$. The reciprocal of $\frac{1}{6}$ is 6. The reciprocal of $\frac{4}{5}$ is $\frac{5}{4}$.

Symbols for Multiplication

The product of 8 and 4 can be written as:

8 times 4 $\quad 8 \times 4 \quad 8 \bullet 4 \quad (8)(4) \quad \begin{array}{r} 8 \\ \underline{\times\ 4} \end{array}$

In algebra, we generally avoid using the $\times$ for multiplication because it could be misinterpreted as the variable *x*, and we cautiously use the symbol $\bullet$ for multiplication because it could be misinterpreted as a decimal point.

Symbols for Division

The quotient of 8 and 4 can be written as:

8 divided by 4 $\quad 8 \div 4 \quad 4\overline{)8} \quad \dfrac{8}{4} \quad$ 8/4

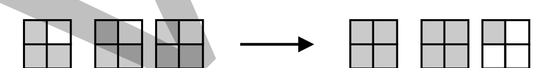
In algebra, the preferred way to show division is with fraction notation.

Visualizing Fraction Multiplication

Grouping (repeated addition)
Thinking about "groups of" is useful when multiplying a whole number times a fraction.

For example, 3 groups of $\frac{3}{4}$ can be written as:

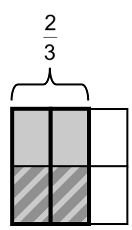
$$3 \bullet \frac{3}{4} = \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{9}{4} = 2\frac{1}{4}$$



Area model
An area model is useful for multiplying proper fractions.

First, $\frac{2}{3}$ of the square is highlighted. Then $\frac{1}{2}$ of that $\frac{2}{3}$ is shaded.

Therefore, the shaded area shows that $\frac{1}{2}$ *of* $\frac{2}{3} = \frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$.



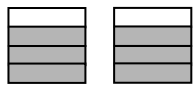
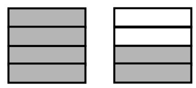
The Multiply-across Rule for Fraction Multiplication

The multiply-across rule for fraction multiplication is: $\frac{a}{b} \bullet \frac{c}{d} = \frac{a \bullet c}{b \bullet d}$

In other words, to multiply two fractions, multiply the numerators of the factors to get the numerator of the product, and multiply the denominators of the factors to get the denominator of the product.

Example 1: $\frac{5}{7} \bullet \frac{3}{4} = \frac{5 \bullet 3}{7 \bullet 4} = \frac{15}{28}$

Example 2: $2\frac{1}{2} \bullet 3\frac{3}{4} = \frac{5}{2} \bullet \frac{15}{4} = \frac{5 \bullet 15}{2 \bullet 4} = \frac{75}{8} = 9\frac{3}{8}$

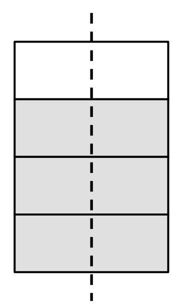
Example: Multiplying Fractions			
Words	Diagrams	Use the multiply-across rule	Use the shortcut notation
A puppy eats two times per day. If the puppy eats $\frac{3}{4}$ cup of kibble at each feeding, how much does it eat in one day?	Start with two groups of $\frac{3}{4}$ (shaded)  Combine the parts  $\frac{3}{4}+\frac{3}{4}=\frac{6}{4}=1\frac{2}{4}=1\frac{1}{2}$	$2\times\frac{3}{4}=\frac{2}{1}\times\frac{3}{4}$ $=\frac{6}{4}$ $=\frac{3}{2}$	$2\times\frac{3}{4}=\frac{\cancel{2}^{1}}{1}\times\frac{3}{\cancel{4}_{2}}$ $=\frac{1\times3}{1\times2}$ $=\frac{3}{2}$

Visualizing Fraction Division as "Divvie Up"

A "divvie up" division problem poses the question:

"How can we divide ___ into ___ equal groups?

Suppose we want to divide $\frac{3}{4}$ cups of grape juice equally among two people. This division problem $\frac{3}{4}\div 2$, can be interpreted as "how can we divide $\frac{3}{4}$ into 2 equal parts?



Let the rectangle represent 1 full cup. It is filled with $\frac{3}{4}$ cups of grape juice.

From the diagram we see that each person will get $\frac{3}{8}$ cup of juice.

Therefore, $\frac{3}{4}\div 2=\frac{3}{8}$.

Visualizing Fraction Division as "Measure Out"

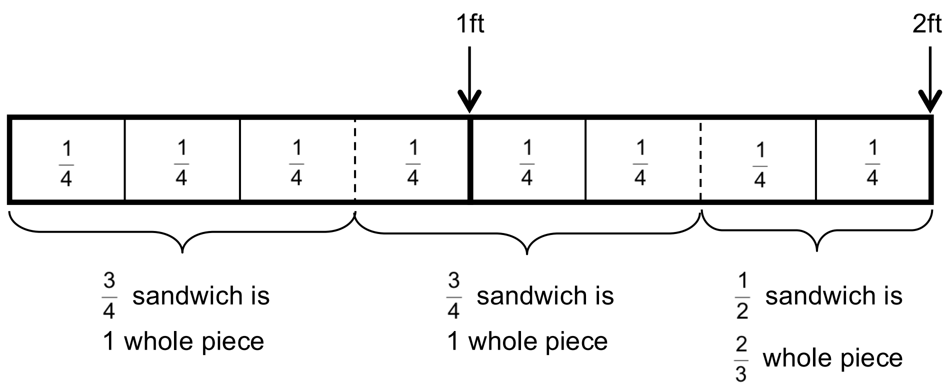
A "measure out" division problem poses the question:

"How many ___ are in ___?"

Suppose a two-foot sandwich is cut into pieces that are $\frac{3}{4}$ foot long each.

- This division problem $2 \div \frac{3}{4}$ can be interpreted as "how many $\frac{3}{4}$ ft. are in 2 ft.?"
- The unit of measure (serving) is $\frac{3}{4}$ ft.
- From the diagram, there are TWO $\frac{3}{4}$ ft. sandwiches in the 2 ft. sandwich.
- From the diagram, there is $\frac{1}{2}$ ft. of sandwich leftover.
- Since $\frac{1}{2} = \frac{2}{3}$ of $\frac{3}{4}$, the leftover represents $\frac{2}{3}$ of the unit of measure.

Therefore, $2 \div \frac{3}{4} = 2\frac{2}{3}$.



Rules for Dividing Fractions

Divide-across

The divide-across rule states that we can divide numerators and divide denominators to find the quotient.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a \div c}{b \div d} \qquad b \neq 0,\ d \neq 0$$

Example 1: $\frac{8}{9} \div \frac{2}{3} = \frac{8 \div 2}{9 \div 3} = \frac{4}{3} = 1\frac{1}{3}$

Notice that for this example, the numbers are "friendly." In other words, 2 divides 8 evenly, and 3 divides 9 evenly.

What if this is not the case? Use equivalent fractions with common denominators.

Example 2: $\frac{1}{4} \div \frac{2}{3} = \frac{3}{12} \div \frac{8}{12} = \frac{3 \div 8}{12 \div 12} = \frac{\frac{3}{8}}{1} = \frac{3}{8}$

In general, we can divide any fraction by any fraction (assuming the divisor is not zero) by finding common denominators first. When we have common denominators, the divide-across rule states:

$$\frac{a}{b} \div \frac{c}{b} = \frac{a \div c}{b \div b} = \frac{a \div c}{1} = \frac{a}{c} \qquad b \neq 0,\ c \neq 0$$

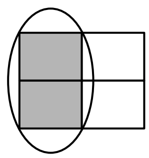
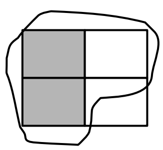
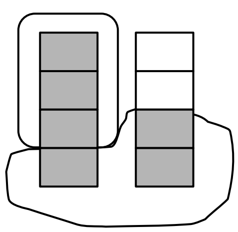
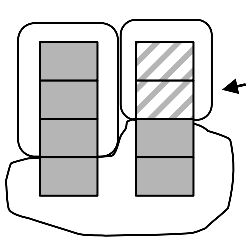
Multiply-by-the-Reciprocal

The multiply-by-the-reciprocal rule states that dividing by a number is equivalent to multiplying by its reciprocal. In other words, to find the quotient, change the divisor to its reciprocal and multiply.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c} \qquad b \neq 0,\ d \neq 0,\ c \neq 0$$

Example: $\frac{8}{9} \div \frac{2}{3} = \frac{\cancel{8}^{4}}{\cancel{9}_{3}} \bullet \frac{\cancel{3}^{1}}{\cancel{2}_{1}} = \frac{4}{3} = 1\frac{1}{3}$

Examples: Dividing Fractions

Words and Diagrams	**Divide-Across**	**Multiply-by-the-Reciprocal**
How many $\frac{1}{2}$s are in $\frac{3}{4}$? 	$\frac{3}{4} \div \frac{1}{2} = \frac{3 \div 1}{4 \div 2}$ "friendly numbers" $= \frac{3}{2} = 1\frac{1}{2}$	$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1}$ $= \frac{3 \times 2}{4 \times 1}$ $= \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$
How many $\frac{3}{4}$s are in $\frac{1}{2}$? 	$\frac{1}{2} \div \frac{3}{4} = \frac{2}{4} \div \frac{3}{4}$ $= \frac{2 \div 3}{4 \div 4} = \frac{2 \div 3}{1}$ NOT "friendly numbers"; use common denominators $= \frac{\frac{2}{3}}{1} = \frac{2}{3}$	$\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3}$ $= \frac{1 \times 4}{2 \times 3}$ $= \frac{4}{6} = \frac{2}{3}$
Christine's dog Barkley eats $\frac{3}{4}$ can of food at each meal. How many meals can Barkley eat with $1\frac{1}{2}$ cans of food? 	$1\frac{1}{2} \div \frac{3}{4}$ $= \frac{3}{2} \div \frac{3}{4}$ $= \frac{6}{4} \div \frac{3}{4}$ $= \frac{6 \div 3}{4 \div 4}$ $= \frac{2}{1} = 2$ Barkley can eat 2 meals.	$1\frac{1}{2} \div \frac{3}{4}$ $= \frac{3}{2} \div \frac{3}{4}$ $= \frac{3}{2} \times \frac{4}{3}$ $= \frac{3 \times 4}{2 \times 3}$ $= \frac{12}{6} = 2$ Barkley can eat 2 meals.
Bobbie's dog Charlie eats $\frac{3}{4}$ can of food at each meal. How many meals can Charlie eat with 2 cans of food? 	$2 \div \frac{3}{4} = \frac{8}{4} \div \frac{3}{4}$ $= \frac{8 \div 3}{4 \div 4}$ $\frac{1}{2}$ can represents $\frac{2}{3}$ of a meal $= \frac{\frac{8}{3}}{1} = \frac{8}{3} = 2\frac{2}{3}$ Charlie can eat $2\frac{2}{3}$ meals.	$2 \div \frac{3}{4} = \frac{2}{1} \times \frac{4}{3}$ $= \frac{8}{3} = 2\frac{2}{3}$ Charlie can eat $2\frac{2}{3}$ meals.

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